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COSMOLOGY AND PARTICLE PHYSICS

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The state of our understanding of cosmology is reviewed from an astrophysical cosmologist point of view with a particular emphasis given to recent observations and their impact. Discussion is then presented on the implications for particle physics.

1 Overview of astrophysical cosmology

By 1970 astrophysicists were already reasonably confident that the universe began as a fireball and the hot universe then cooled as it expanded. The 2.7K radiation and the 25% helium abundance are among the strongest fossil evidence supporting this scenario. This is beautifully described in Weinberg's book *The First Three Minutes*[1] published in 1977. It is remarkable that the work over the last three decades has not found anything which would invalidate this view, but only strengthened the evidence for it. Furthermore, the subsequent research has made it possible to delineate the story beyond the first three minutes up to the present, which was missing in Weinberg's book, namely the story that is dominated by the formation of cosmic structure. Attempts to understand cosmic structure formation have greatly enriched cosmological tests both for structure formation itself and for the evolution of the universe as a background to the structure. Successful results of a number of key tests lead us to conclude that we are approaching understanding of the evolution of the universe and cosmic structure.

Cosmology today is based on three paradigms: (i) the hot Big Bang and the subsequent Friedmann-Lemaître expanding universe, (ii) the universe today being dominated by cold dark matter (CDM)[2], and (iii) the presence of inflation in some early period[3]. (ii) is still hypothetical and (iii) is even more so. Yet, we cannot construct a reasonable model of the universe without the aid of these two concepts. The most important implication of inflation is the generation of density fluctuations over superhorizon scales, the presence of which is firmly established by the observations of the cosmic microwave background (CMB) by the COBE satellite[4].

The formation of structure basically reads as follows. At some early epoch density fluctuations are generated adiabatically. The most promising idea ascribes the origin to thermal fluctuations of the Hawking radiation in the de Sitter phase of inflation, and these fluctuations are frozen into classical fluctuations in the inflation era[5]. The observed fluctuations are close to Gaussian noise with their spectrum usually represented as

$$P(k) = \langle|\delta_k|^2\rangle \propto k^n \ , \tag{1}$$

where n is close to unity. This noise is amplified by self gravity in an expanding universe[6]. The fluctuations grow only when the universe is matter dominated and the scale considered (i.e., $2\pi/k$) is within the horizon. Therefore, the perturbations are modified by a scale dependent factor as they grow

$$P(k,z) = D(z)k^nT(k) \ , \tag{2}$$

where $D(z)$ is the growth factor as a function of redshift, and the transfer function $T(k) \sim 1$ for small k and $\sim k^{-4}$ for large k. The transition takes place at $k \simeq k_{eq} \simeq 2\pi/ct_{eq}$, where

$$ct_{eq} = 6.5(\Omega h)^{-1} h^{-1}\text{Mpc} \qquad (3)$$

is the horizon scale at matter - radiation equality[7]. $T(k)$ is called the transfer function. Hence, the universe acts as a low-pass gravitational amplifier of cosmic noise. The amplitude of the fluctuations that enter the horizon is nearly constant $(n \simeq 1)$[8] and is of the order of 10^{-5}. The small-scale fluctuations become non-linear at $z \simeq 10 - 20$, and the first objects form from high peaks of rare Gaussian fluctuations. As time passes, lower peaks and larger scale fluctuations enter the non-linear regime, and eventually form gravitationally bound systems which decouple from the expansion of the universe. We call this state 'collapsed'. At the present epoch ($z = 0$) objects larger than $\sim 8h^{-1}$Mpc are still in the linear regime.

The fluctuations and their evolution are described by a single function of the power spectrum $P(k)$ scaled to today, with the normalisation represented by rms mass fluctuations within spheres of radius of $8h^{-1}$Mpc:

$$\sigma_8 = \langle(\delta M/M)^2\rangle^{1/2}|_{R=8h^{-1}\text{Mpc}} \qquad (4)$$

$$= \int_0^\infty 4\pi k^2 dk |\delta_k|^2 W(kR) \ , \qquad (5)$$

where $W(kR)$ is called the window function. The fluctuations (adiabatic fluctuations) before recombination epoch ($z \sim 1000$) are imprinted on the CMB, and they are conveniently represented by multipoles of the temperature field as,

$$\langle(\Delta T/T)^2\rangle = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell \ . \qquad (6)$$

For illustration the power spectrum is shown in Figure 1.

The empirical match of the power spectra estimated from large scale galaxy clustering and from CMB (COBE), which generically differ by a factor of 10^5, has brought us confidence that we are working with the correct theory[10,11]. Here the CDM hypothesis plays a crucial role. Without CDM this matching is impossible, or more precisely we do not know any alternatives yet.

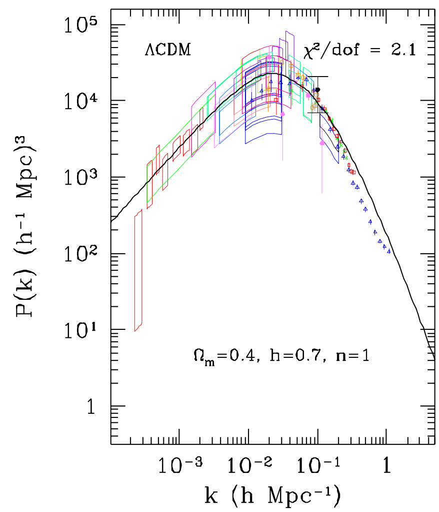

Figure 1. Power spectrum derived from large scale clustering of galaxies (shown with data points) and CMB temperature fluctuations (shown with boxes). The curve is the model power spectrum of a flat CDM universe with a cosmological constant. The figure is taken from Gawiser and Silk [9].

As fluctuations grow, they enter a non-linear regime. This phase was first extensively studied by the use of N body simulations. The statistical results of simulations are very well described with an approach called the Press-Schechter formalism[12], in which Gaussian fluctuations, when they exceed some threshold[13,7], follow nonlinear evolution as described by a spherical collapse model[14]. This allows us to treat statistical aspects of non-linear growth in an analytic way. Figure 2 shows the mass fraction of collapsed objects with mass $> M$ as a function of redshift z.

Whether the collapsed object forms a brightly shining single entity (galaxy) or an assembly of galaxies depends on the cooling time scale ($t_{\rm cool}$) compared to the dynamical scale, $t_{\rm dyn} \sim (G\rho)^{-1/2}$ [15]. For $t_{\rm cool} < t_{\rm dyn}$, the object cools and shrinks by dissipation and stars form, shining as a (proto)galaxy. Otherwise, the object remains a virialised

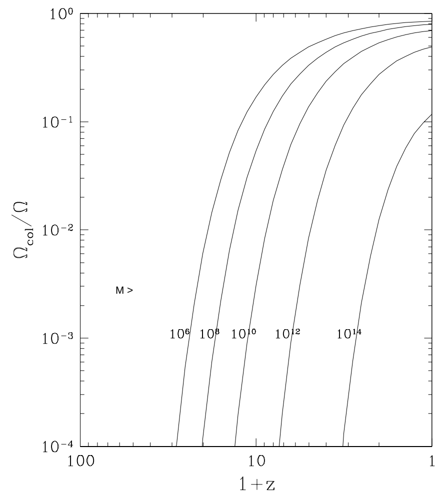

Figure 2. Fraction of gravitationally collapsed objects with mass greater than a given value M (in solar mass units) as a function of redshift $1+z$. The calculation uses the Press-Schechter formalism and assumes the CDM model with parameters $\Omega = 0.3$, $\lambda = 0.7$ and $\sigma_8 = 0.9$.

cloud, and is observed as a group or a cluster of galaxies. In the latter case only gravity works efficiently, so that the system is sufficiently simple to serve as a test for gravitational clustering theory. Galaxy formation is very complicated due not only to the action of the cooling process, which eventually leads to star formation, but also to feedback effects, such as UV radiation and supernova winds from stars. We expect that the first galaxies form at around $z \sim 10$. The period between $z \sim 1000$ and the epoch of first galaxy formation constitutes a dark age of cosmological history. Observationally, the highest redshift securely measured is $z = 5.8$ for a quasar[16]. Even higher redshift galaxies have been reported, though the redshift measurement is not as secure as for quasars. How galaxies formed and evolved is the most important arena for astrophysical cosmologists today, both theoretically and observationally. I omit to discuss this subject in this talk, however, since it does not seem to give us insights into particle physics; it is entirely a world of astrophysics.

Before concluding this section I would emphasise that crucial cosmological tests can be made by the convergence of cosmological parameters, notably the Hubble constant, H_0, the cosmic matter density ρ in units of the Einstein-de Sitter (EdS) closure value $\rho_{\rm crit}$, Ω, and the cosmological constant or vacuum energy density in units of $\rho_{\rm crit}$, $\lambda = \Lambda/3H_0^2 = \rho_V/\rho_{\rm crit}$; $\Omega + \lambda = 1$ defines a flat (zero curvature) universe. H_0 is often represented by $h = H_0/100$ km s^{-1}Mpc^{-1}.

2 Hubble constant

The Hubble constant, which has dimension of inverse time, sets the scale of the size and age of the Universe. Recent efforts have almost solved the long-standing discrepancy concerning the extragalactic distance scale; at the same time, however, significant uncertainties are newly revealed in the distance scale within the Milky Way and to the Large Magellanic Cloud (LMC), the mile stone to the extragalactic distance.

The global value of H_0 was uncertain by a factor of two for several decades. The discovery of a few new distance indicators around 1990 has made possible an estimation of the systematic error for each indicator by cross-correlating the resulting distances (For a review of the methods, see [17]). This greatly enhanced the reliability of Hubble constant determinations. The error shrunk, notwithstanding there was still a dichotomous discrepancy between $H_0 = 80$ and 50 depending on the method used[18]. This meant that there still existed systematic effects that were not understood. The next major advance was brought with the refurbishment mission of HST, which enabled one to resolve Cepheids in galaxies as distant as 20 Mpc (HST Key Project[19]). This secured the distance to the Virgo cluster and tightened the calibrations of the extragalactic distance indicators, and resulted in $H_0 = (70-75) \pm 10$, 10% lower than the 'high value'[20]. Another important contribution was the discovery that the maximum brightness of type Ia supernovae (SNeIa) is not an absolute standard candle, but correlates with the decline rate of brightness[21], along with the direct calibration of the maximum brightness of several SNeIa with HST Cepheid observations[22]. The resulting H_0 was 64 ± 3, appreciably higher than 50. These results nearly resolved the long-standing controversy.

Extragalactic distance ladders are calibrated with the Cepheid period-luminosity relation. The majority of observations are provided by the HST-KP, whereas those of SNIa host galaxies are given by Saha, Sandage and collaborators. It was found by the HST-KP group that the Saha-Sandage distances that calibrate the SNIa brightness are all systematically longer by 5-10%[23] for different reasons for different galaxies[24]. The result of HST-KP is confirmed by [25].This makes the Hubble constant from SNeIa 69$\pm$4.

Another distance indicator that allows an accurate estimation is surface brightness fluctuations. The current best result based on 300 galaxies is $H_0 = 77 \pm 7$, or 74 ± 4 with a model of the peculiar velocity flow from galaxy density distributions[26]. Taking SN and SBF results we may conclude $H_0 = 71 \pm 7$ (2σ)[27], in agreement with the 2000 summary of the HST-KP group[28]. HST-KP group slightly updated H_0 in their later report[29]: $H_0 = 74 \pm 7$. Further reduction of the error needs accurate understanding of interstellar extinction corrections and metallicity effects, which is by no means easy.

All the methods mentioned above use distance ladders and take the distance to the Large Magellanic Cloud (LMC) to be 50 kpc (distance modulus $m - M = 18.5$) as the zero point. Before 1997 few doubts were cast on this. With the exception of RR Lyraes, the distances have converged to $m - M = 18.5 \pm 0.1$, i.e., within 5% error, and the RR Lyr discrepancy was blamed on its larger calibration error. The work over the last three years, notably that by the Hipparcos astrometric satellite (ESA 1997), revealed that the distance to LMC is not as secure as has been thought. The current estimate of the LMC distance varies from 43 to 53 kpc. This means that the Hubble constant is uncertain by a factor $0.95-1.15$ [27]. Leaving this uncertainty I conclude the Hubble constant to be

$$H_0 = 71 \pm 7 \times \frac{1.15}{0.95} \text{ km s}^{-1}\text{Mpc}^{-1} . \quad (7)$$

Cosmic age

The estimate of the age of the universe uses the position of turn off from main sequence tracks in the HR diagram of globular clusters. The age thus estimated also turned out to be more uncertain than had been thought. The major elements of uncertainties are the zero point of RR Lyr (20%) and the interpretation as to the formation of globular clusters, whether their heavy element abundances indicate the formation epoch or their formation was coeval independent of the heavy element abundance (20%). The first uncertainty is related to that of the LMC distance: the calibration giving a long distance to LMC gives a shorter age of clusters. The minimum of the estimated age is 12$\pm$1 Gyr[30] and the maximum is 18$\pm$2 Gyr, where $\pm$ reflects errors other than are discussed here; see [27].

3 Ω and Λ

The mass density parameter Ω, as measured in units of the critical density, controls the cosmic structure formation. From the cosmic structure formation point of view the role of the cosmological constant λ is subdominant: it partially compensates the slow speed of structure formation in a low density universe.

Whether Ω and λ inferred from the geometry of the universe or dynamics agree with those with the aid of structure formation models serves as an important cosmological test not only for the validity of the Friedmann universe but also for the model of cosmic structure formation.

Determinations of Ω and λ which can be carried out without resorting to specific structure formation models are:
(1) $H_0 - t_0$ matching using $t_0 = H_0^{-1} f(\Omega, \lambda)$, which gives $\Omega < 0.8 - 0.9$. This at least excludes the Einstein-de Sitter ($\Omega = 1$) universe.
(2) Luminosity density and the average mass to light ratio of galaxies, $\Omega = \mathcal{L}\langle M/L\rangle/\rho_{\rm crit}$. This gives $\Omega = 0.2 - 0.5$. A slightly larger value compared to those in the literature is due to a correction for unclustered components[31].
(3)* Cluster baryon fraction, as inferred from X-ray emissivity[32,33] or the Zeldovich-Sunyaev effect[34]. This should match with the global value Ω_b/Ω, where Ω_b is the baryon density inferred from primordial nucleosynthesis.
(4) Peculiar velocity - overdensity relation, $\nabla \cdot v_p = -H_0\Omega^{0.6}\delta$, a direct derivative from gravitational instability theory[35]. The result of this test is still grossly controversial; the estimate varies from $\Omega = 0.2$ to 1.
(5) Type Ia supernova Hubble diagram, which measures the luminosity distance, $d_L = d_L(z; \Omega, \lambda)$. The result is summarised as $\Omega = 0.8\lambda - 0.4 \pm 0.4$ [36,37].
(6) Gravitational lensing frequency. The image of distant quasars occasionally splits into two or more images due to foreground galaxy's gravitational potential. The frequency is sensitive to λ, while it is nearly independent of Ω. The current limit[31] is $\lambda < 0.8$.

Determinations that depend on specific structure formation models are:
(7) Shape parameter of the transfer function. The break of the transfer function depends on the shape parameter $\Gamma \simeq \Omega h$, and this is estimated from large scale galaxy clustering, as $\Gamma + (n-1)/2 = 0.15 - 0.3$ [38,39]. This means $\Omega \simeq 0.35$.

(8)* Matching of the cluster abundance with the COBE normalisation. The cluster abundance at $z \approx 0$ requires the rms mass fluctuation to satisfy[40] $\sigma_8 \approx 0.5\Omega^{-0.5}$. σ_8 is also accurately determined by the large-scale fluctuation power imprinted in the CMB with the aid of eq.(2). The result depends on the power n, but notwithstanding $\Omega > 0.5$ cannot be reconciled with observations unless n is largely deviated from unity[11].

(9)* Multipoles of CMB temperature fields: the position of the acoustic peak is roughly proportional to $\ell_1 \approx 220[(1-\lambda)/\Omega]^{1/2}$. A compilation of C_ℓ measurements favours $\Omega + \lambda \approx 1$ [41,42].

(10) Evolution of cluster abundance[43]. The evolution of rich cluster to $z \approx 0.5 - 0.8$ is more sensitive to σ_8, and one can separately determine σ_8 and Ω by studying the evolution of the cluster abundance. The results, however, are at present dichotomous: $\Omega = 0.2 - 0.45$ [44] and ≈ 1 [45].

The items with asterisks will be revisited in the next section. Among the ten tests, (5), (6) and (10) are particularly important tests for the cosmological constant. We have omitted well-known 'classical tests' such as the number count of galaxies, angular-diameter redshift relation, and the redshift distribution of galaxies, since these tests depends on the galaxy evolution, the understanding of which is still far from complete.

The conclusion we can draw from this list is a gross convergence to $\Omega = 0.2 - 0.5$ and indications for a finite λ (SNIa Hubble diagram and CMB multipoles). We shall see in the next section that these conclusions are corroborated by the new data of CMB observations, as seen in Figure 6 below.

4 Impacts of the new CMB experiments

The hot news of this year is the data release of two high resolution CMB anisotropy experiments using balloon flights. One is BOOMERanG[46] flight in Antarctica observing the southern sky and the other is MAXIMA[47], a North American flight exploring the northern sky. The two experiments have the beam sizes of the order of $10'$, significantly smaller than that of COBE, and explore CMB multipoles for $\ell = 26-625$ and $36-785$, respectively. These data cover both first and second peak regions, and MAXIMA marginally reaches the third peak region. The one sigma error is about 10−20% for each data point, and normalisation errors are 10% (BOOMERanG) and 4% (MAXIMA). BOOMERanG gives more restrictive information for low ℓ.

BOOMERanG presents a map of CMB sky at 90, 150, 240 GHz (at which CMB is supposed to dominate) and 400 GHz (at which dust emission dominates). The maps at the first three frequencies show patterns in a remarkable agreement, verifying that the fluctuations are indeed intrinsic to CMB. On the other hand, the map at 400 GHz shows a completely different pattern, correlated well with interstellar dust emission known from infrared observations[48]. The 400 GHz map agrees with a map obtained from the difference of the two maps at 240 and 150 GHz.

The multipoles extracted from two observations, which observe the opposite sides of sky with respect to the Galactic plane, show an excellent agreement with each other (except for one point at $\ell \approx 140$) once we shift the data within the normalisation errors (see Figure 3). These data are also quite consistent with the earlier experiments if the average is taken over the data with large errors and large scatters. The two experiments have brought important improvement in the accuracy, which allows us to derive solid conclusions on the cosmological parameters from CMB.

A number of extensive analyses followed the data release, and the conclusions, while they are expressed in different ways, agree with each other [49−54]. Most authors use a

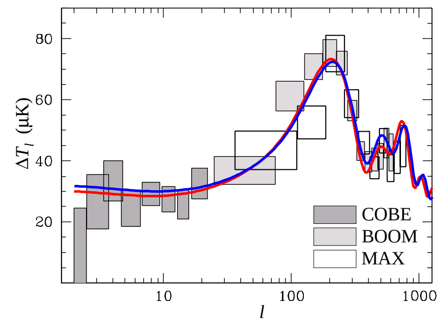

Figure 3. CMB multipoles $\Delta T_\ell = \sqrt{\ell(\ell+1)C_\ell/2\pi}$ from BOOMERanG and MAXIMA experiments (the normalisations are shifted within one sigma calibration error), together with the COBE 4 year data. The curves show the prediction of the CDM structure formation model. The thick solid curve represents the model that satisfies the joint constraint: $\Omega = 0.35$, $\lambda = 0.65$, $h = 0.75$, $\Omega_b h^2 = 0.023$ and $n = 0.95$. The grey curve is the model that is a good fit to CMB alone: $\Omega = 0.3$, $\lambda = 0.7$, $h = 0.9$, $\Omega_b h^2 = 0.03$ and $n = 1$. Note a high baryon abundance for the latter. Figure is taken from Hu et al.[53].

general likelihood analysis in multiparameter (typically 7 parameter) space imposing varieties of prior conditions, while Hu et al. [53] developed a parametric approach to make correlations among parameters more visible.

The major conclusions we can derive from these CMB data alone are:

(1) The position of the first peak is securely measured to be $\ell = 206 \pm 6$. This means that the universe is close to flat. See Figure 4. See also (9) of section 3.

(2) The spectral index is close to unity: $n = 1 \pm 0.08$.

(3) The height of the second peak is significantly smaller than was expected from the standard set of cosmological parameters, pointing to a baryon abundance that is higher than is inferred from primordial nucleosynthesis[55]. The best fit requires $\Omega_b h^2 \simeq 0.03$ compared with $\Omega_b h^2 < 0.023$ (95% confidence) from nucleosynthesis. The CMB data are consistent with the upper limit from nucleosynthesis only at a 2 sigma level with a red tilt of the perturbation spectrum:

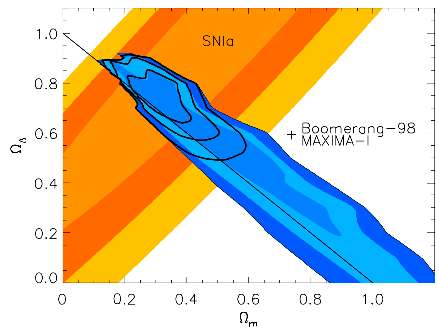

Figure 4. Constraints on the $\Omega - \lambda$ plane derived from CMB multipoles and a type Ia supernova Hubble diagram, taken from Jaffe et al.[54].The labels are: $\Omega_m \equiv \Omega$ and $\Omega_\Lambda \equiv \lambda$. The three levels of shading mean 1, 2 and 3 sigma. The contours show the joint constraint. The straight line indicates flat universes.

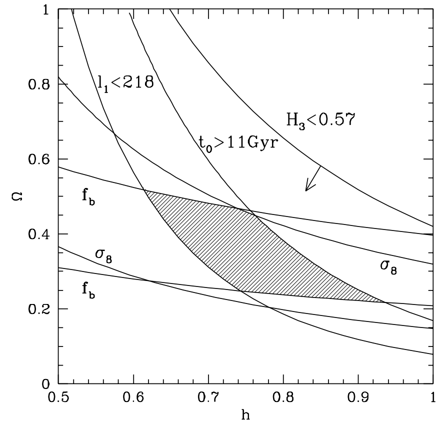

Figure 5. Constraints on the $h - \Omega$ plane derived from the new CMB experiments. ℓ_1 stands for the position of the first peak, H_3 is the ratio of the heights of the third peak to the second, the curves labelled by 'σ_8' are obtained by the match of CMB data with the cluster abundance, and those with 'f_b' are the constraint from the CMB and the cluster baryon fraction. Cosmic age $t_0 > 11Gyr$ is also plotted. The curves are taken from [53].

$$0.85 < n < 0.98. \tag{8}$$

The 2 σ lower limit derived from CMB is $\Omega_b h^2 > 0.019$.

(1) is one of the most straightforward constraints derived from CMB, and based only on geometry and acoustic physics. In the flat universe the derived constraint agrees with $t_0 < 13.5$ Gyr.

(1) and (2) are taken to be a strong support for standard cosmology based on the CDM dominance of matter and adiabatic density perturbations. They also support the idea of inflation as the origin of density fluctuations; fluctuations from pure defects (cosmic strings, textures) are excluded. On the other hand, (3) indicates marginal consistency with the baryon abundance in our current standard understanding; inconsistency would become acute if the accuracy of the data increases with the central values fixed.

The CMB data alone are not sufficient to uniquely determine the cosmological parameters. When they are supplemented with the information on large scale structure (either (7) or (8)), we are led to:

$$\Omega = 0.4 \pm 0.2, \quad H_0 = 75 \pm 15, \quad \lambda = 1 - \Omega {}^{+0.2}_{-0.1}. \tag{9}$$

Figure 5 presents the constraints derived from the CMB either directly or indirectly with the aid of external constraints (Ω_b from nucleosynthesis, cluster abundance, and cluster baryon fraction) in the $h - \Omega$ plane. The allowed parameter region agrees with what are discussed in section 3, as shown in Figure 6, where it is shown together with the constraints independent of CMB. The significance is that the cosmological parameters derived via the structure formation model agree with each other but also with model-independent analysis, corroborating our understanding of cosmology and structure formation. This argument will be elaborated (or falsified) upon the completion of the currently on-going CMB experiments, DASI, CBI and MAP.

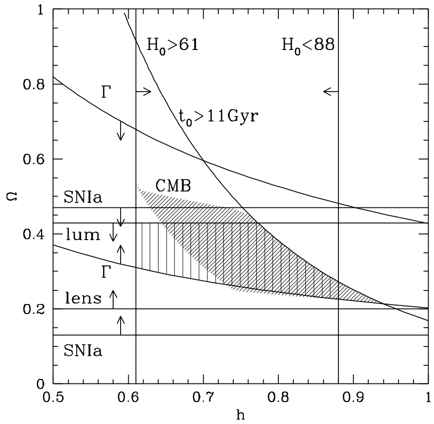

Figure 6. Summary of constraints shown in the $h-\Omega$ plane. Constraints shown by lines correspond to (1), (2), (5)-(7) of §3 and the range of the Hubble constant in §2. The allowed region derived from CMB (corresponding to (3), (8) and (9) of §3 and cosmic age) is shown by thick shading, while those derived independent of CMB are indicated by light shading.

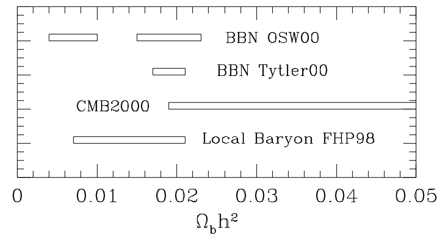

Figure 7. Baryon abundances inferred from CMB (discussed above), nucleosynthesis (BBN)[56,59],and accounting the local baryon distribution[60].

5 Matter content of the universe

5.1 Baryons

The total baryon abundance, as represented by the baryon to photon ratio $\eta = n_b/n_\gamma$, is inferred from nucleosynthesis of light elements d, ^{4}He, and ^{7}Li. With $T = 2.728$K, we have $\Omega_b h^2 = 0.00367(\eta/10^{-10})$. A recent review of Olive et al.[56] gives two solutions $0.004 < \Omega_b h^2 < 0.010$ and $0.015 < \Omega_b h^2 < 0.023$ as 2 sigma allowed ranges. The major change over the last five years is the new input from deuterium abundance measured for Lyman α absorbing clouds (Lyman limit systems) at high redshift, and a higher He abundance reported by Izotov & Thuan[57]. Deuterium lines are observed for five Lyman limit systems; three of them gives low deuterium abundance, while the other two (including the one observed at the first time[58]) give high abundance. Assuming the lower abundance to represent the true value, Tytler takes an average of the three and concludes D/H$= 3.4 \pm 0.25 \times 10^{-5}$, which turns into $0.019 < \Omega_b h^2 < 0.021$ [59]. The primordial He abundance of Izotov & Thuan is $Y_p = 0.245 \pm 0.002$, which is compared with the traditional value 0.235 ± 0.003. The difference primarily arises from the use of different calculation of the helium recombination rates and different corrections for collisional excitation than were used in the past. So the difference is of systematic nature rather than due to errors in the observations. It seems that more thorough studies of systematic errors are needed for the extraction of primordial elemental abundances.

The relative height of even to odd harmonic peaks of CMB multipoles is sensitive to the baryon abundance, and the upper limit from BOOMERanG and MAXIMA clearly rules out the low baryon option, but also it is marginally consistent with the high baryon option (see Figure 7).

We note that only 10% of baryons are frozen in stars which are visible in optical observations $\Omega_{\rm star} = 0.004 \pm 0.002$ at $h = 0.7$; baryons in the hot gas component which is visible through X-ray emission is a similar amount. It is inferred that the rest is present around the galaxies in the form of warm gas that is not easily detectable. It seems that the high baryon option is barely consistent with the amount which is obtained by adding all budget list for baryons[60].

5.2 Dark matter

The presence of 'non-baryonic' dark matter is compelling. Among others the most important evidence is (i) the mismatch of $\Omega \approx 0.3$

with Ω_b from nucleosynthesis by one order of magnitude, and (ii) the matching of fluctuations in CMB at $z = 1000$ with those inferred from large scale structure at $z \approx 0$. If dark matter is baryonic and couples to photons, this agreement of (ii) is completely lost: yet we do not know any theories that give a correct matching between CMB and large scale structure without the aid of the CDM dominance.

A promising candidate of the dark matter is weak interacting massive particles as a relic of the hot universe, as discussed widely by particle physicists (see talks by Drees, Olive and Arnowitt[61]). If these particles were in thermal equilibrium the dark matter density would be $\Omega \sim 3 \times 10^{-27} \langle \sigma_{\rm ann} v_{\rm rel} \rangle^{-1}$, where average is taken over thermal distributions at the epoch of the decoupling of dark matter, which is about $T \sim 0.05 m_{\rm dark}$. The important fact is that the desired amount of dark matter is obtained with physics of typical weak interaction scale: $\langle \sigma_{\rm ann} v_{\rm rel} \rangle^{-1} \sim G_F^2 T^2 \sim 3 \times 10^{-26}$ cm^{3}s^{-1} for $m_{\rm dark} \sim 100$ GeV. This makes the lowest supersymmetric particle a natural candidate (see [62] for a review). The current most promising candidate is the neutralino that is a mixture of the photino, zino and Higgsino (tg$\beta > 3$, $M_\chi > 50$ GeV)[63,64].

Massive neutrino.

We are now convinced that neutrinos are massive. The mass density corresponding to $m_\nu \simeq 0.05$ eV is $\Omega_\nu \simeq 0.001$. This is the lower limit and the mass density could be larger if neutrino oscillation experiments are observing the difference of two or more degenerate neutrino masses. From the view point of cosmology, neutrinos can no longer be a candidate for the dominant component of dark matter. The universe dominated by neutrinos does not give correct structure formation, due to free streaming in the early universe that smooths out small scale fluctuations. It has been discussed within the EdS universe that a small admixture ($\sim 20\%$) of light neutrino component would enhance relatively the large scale power required by observation[65]. In a low matter density universe, however, sufficient large scale power is expected without massive neutrinos, and addition of massive neutrinos only disturbs the CMB cluster abundance matching[66,67,68]. Figure 8 shows the effect of massive neutrinos on the power spectrum. The effect on small scale is apparent even if the neutrino mass is as small as 1 eV or less. Accepting the conventional baryon abundance upper limit, the CMB-cluster abundance matching leads to

$$\sum_i m_{\nu_i} < 4eV \tag{10}$$

at 95 % C.L.[53]. A stronger limit is derived if mass density is smaller, say $\Omega < 0.4$ [68].

Strongly interacting dark matter?

The possibility is recently discussed that dark matter might be strongly interacting. The motivation is that N body simulations with CDM models predict halo profiles more singular at the centre of the core and more small-scale objects than are observed[69]. This problem would be solved [a] if dark matter is strongly interacting[70] (see [71], however) or it undergoes annihilation[72]. Either scenario requires the cross section to be of the order of strong interaction. While this problem would offer another arena for particle physics, it seems much more surprising if particles with such properties exist in nature. I would like to ascribe it to our incomplete understanding of astrophysics at small scales which is certainly more complicated than large-scale dynamics.

5.3 MACHO

`MACHO`'s (massive astrophysical compact halo objects) are a possible form of cold dark matter. These objects are collisionless, do not couple to photons, and have no cutoff scale for perturbations; so they would satisfy

[a] Warm dark matter is another possibility[73].

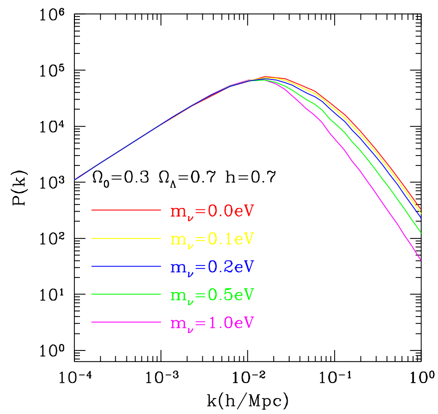

Figure 8. Effect of massive neutrinos on the power spectrum. The three species of neutrinos are assumed to have equal mass, *i.e.*, $\sum_i m_{\nu_i}$ is three times the value indicated by labels. The curve on the top is with $m_\nu = 0$.

requirements for structure formation. The most likely candidate is aborted stars or stellar remnants. This is nearly the only possibility for baryonic dark matter, although the nucleosynthesis constraint on total baryonic matter still have to be evaded.

The novel feature of this form of dark matter is that it would cause gravitational lensing when it passes through the line of sight to background stars. The deflection angle is too small to be observed, but it causes a large magnification of the flux if it crosses the line of sight with a small impact parameter[74]. The duration of 'flash' is $\approx 70(M/M_\odot)^{1/2}$ days for background stars in LMC. Possible sources of confusion are variable or flare stars that would mimic the effect. This confusion, however, can be avoided by observing events with more than one wavelength passband, since the stellar variation is associated with the variation of temperature and the variations in different colour bands are not identical, in contrast to the gravitational lensing which is purely geometrical.

Observations have been made mainly by two groups. The MACHO collaboration conducted the observation for 5.7 years and found $13-17$ events towards LMC which are compared with 70 if the halo consists entirely of `MACHO`[75]. The EROS group found 3 events compared to 27 expected[76]. It is not clear whether detected '`MACHO`'s are a part of the halo dark matter or not. From the time duration of the events these objects should have mass between 0.1 to $1M_\odot$, the mass typical of ordinary stars. The MACHO collaboration concludes the fraction f of `MACHO` to the total halo matter to be about 0.2, while EROS group only quotes $f < 0.4$ (at 95% C.L.) as an upper limit. The definitive and important conclusions are that (a) objects with a mass range of $10^{-7} - 10^{-2}M_\odot$ cannot be a candidate of dark matter, and (b) `MACHO`, if any, accounts only for a minor fraction of dark matter. We add a remark that the lensing cannot be associated with main sequence stars. The number is also too large if they are to be ascribed to white dwarfs (see [77] in this connection).

6 Theory of cosmological constant

We have now almost compelling evidence for a non-vanishing cosmological constant. The traditional problem is why the cosmological constant is so small, but now we are faced with anther problem why it is non-vanishing and it is close to matter density. Traditional attempts to understand the first problem are summarised by Weinberg[78]. Many further attempts have recently been made, but the situation seems still far from the solution. Here I quote three categories of the attempts.

(1) Time varying cosmological constant (Quintessence).

The cosmological constant is decreasing with time, as realised, for example, by a scalar field (quintessence field) slowly evolving down a potential. In this view, we can start with a large cosmological constant. The problem is why vacuum energy density and matter density are approximately equal. Peebles & Ratra[79] have considered the potential of the form,

$$V \sim M^{4+\alpha}\phi^{-\alpha} \ . \tag{11}$$

This model was studied more recently by Zlatev, Wang & Steinhardt[80] who showed that the ϕ field rolls down this potential works as an attractor-like solution to the equation of motion, in the sense that the field and its derivative approach a common evolutionary track for a wide range of initial conditions ('tracker solution'). With an appropriate choice of M we get $\rho_V \sim \rho_m$. The weak points are that one has to tune M to give $\rho_V \sim \rho_m$, and an addition of a constant term spoils the desired behaviour. Armendariz-Picon et al.[81] further developed a model so that negative pressure automatically becomes effective after the epoch of matter-radiation equality. This model needs a modification of the kinetic term (k-essence).

(2) Use of exact symmetry.

With exact supersymmetry the cosmological constant vanishes. In supersymmetry theories in 4 space-time dimension, however, the disparity of fermion masses and their boson partners means breaking of supersymmetry, which inevitably results in a positive cosmological constant of the order of the supersymmetry breaking scale. This constant may be cancelled by some counter term in supergravity theory with an extreme fine-tuning, but this does not solve the problems posed above.

Witten observed in 3 dimensional theory that the disparity arises between fermion and boson masses when matter interacts with gravity, while supersymmetry is maintained[82]. Namely, $Q_{\rm SUSY}|0\rangle = 0$ and $[Q_{\rm SUSY}, H] = 0$, so that the cosmological constant vanishes. When this model is embedded into supergravity theory having a dilaton field, the compactified dimension is stretched to the fourth dimension in the strong coupling limit of the dilaton coupling, making the theory full four dimensional.

Once we have zero cosmological constant we must consider a mechanism to generate a small vacuum energy. An idea is to consider ultra mini-chaotic inflation. If the potential for a scalar field is very flat with the mass of the order of Hubble constant, the initial state at $\phi \approx M_{\rm pl}$ still remains at the same value due to the Hubble viscosity, giving a very small vacuum energy. Such a minuscule mass may be generated by electroweak instanton effects. This proposal in [83] is justified by an explicit calculation within a supergravity model[84], which yields $m \sim G_F^{5/4} m_q^{5/2} M_{\rm SUSY}^{3/2} M_{pl}^{-1/2} \exp(-4\pi^2/g_2^2)$ (G_F is the Fermi constant, m_q the quark mass, $M_{\rm SUSY}$ is the SUSY breaking scale, and g_2 is the SU(2) gauge coupling constant). This gives about the correct order of magnitude for the cosmological constant.

(3) Anthropic principle.

If our universe is one member of an ensemble, and if the vacuum density varies among the different members of the ensemble, the value observed by us is conditioned by the necessity that the observed value should be suitable for the evolution of our Galaxy and of intelligent life. This argument is called the anthropic principle, as explicitly stated by Carter[85]. (More discussion will be made in section 8.) An application of the anthropic principle to the vacuum energy is first discussed by Weinberg[86].

The argument is that galaxies would not have formed if vacuum energy were larger than some critical value, since vacuum energy, providing a repulsive force, hinders evolved perturbations from collapsing into galaxies. The condition is roughly expressed as $\rho_V < \rho_m$ at $1 + z \sim 4 - 5$, where galaxies formed. This translates to $\rho_V < 100\rho_m$ today, a dramatic narrowing of the allowed range for the cosmological constant.

Some authors further elaborated the argument by adopting the hypothesis called 'principle of mediocrity' which says that we should expect to find ourselves in a big bang that is typical of those in which intelligent life is possible[87]. A working assumption to calculate the probability of civilisation is that it is proportional to the number of baryons frozen into a galaxy, and the *a priori* probability of a universe having ρ_V, $P(\rho_V) \simeq$constant. Calculations are made for galaxy formation according to the formalism described in section 1. The result depends on further input assumptions, but it generally is not too far from the value with which we actually live[88]. For instance Martel et al. obtained the probability of finding ourselves in a universe with $\lambda < 0.7$ to be 5-12%. Whether $P(\rho_V) \simeq$constant is realised is investigated within scalar field theory in [89].

7 Theory of inflation

Cosmological inflation gives a universe a number of desirable features. The most important among them is the generation of density fluctuations over superhorizon scales. Quantum particle creations in the de Sitter phase generate thermal fluctuations corresponding to the effective temperature $T = \hbar H/2\pi$ (H being the expansion rate) and they are frozen into classical fluctuations when the scale considered goes outside the horizon in the inflation era. Inflation is the only known mechanism that can generate empirically viable fluctuations.

Theory of inflation assumes the presence of one or more scalar fields, called inflatons, which obey the field equation,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \ . \tag{12}$$

Inflation takes place if the second term that works as viscosity is large enough so that roll down of the state is sufficiently slow: This slow roll regime is realised when

$$\epsilon = \frac{M_{pl}^2}{16\pi}(V'/V)^2 \ll 1, \quad \eta = \frac{M_{pl}^2}{8\pi^2}(V''/V) \ll 1. \tag{13}$$

There are several points to be satisfied to make the model observationally viable:

(1) $\Omega + \lambda = 1$

(2) $N_{e-\mathrm{fold}} \geq 50$, where N is the logarithmic ratio of the scale factors before and after inflation.

(3) $V^{3/2}/M_{pl}^3 V' = 4\times 10^{-6}$ as required by the COBE observation, $Q = 2 \times 10^{-5}$. This is a crucial condition to have successful structure formation and the presence of ourselves.

(4) The spectral index $n \approx 1$. The CMB observations indicate $n = 1 \pm 0.15$ at 95% C.L. If we require the consistency with primordial nucleosynthesis, only red tilt is allowed as shown in eq.(8). The theory of inflation predicts $n = 1 - 6\epsilon + 2\eta$ which is close to, but *not* quite unity. Simple (one field) models of inflation predict $n < 1$ (red tilt), whereas $n > 1$ requires a more complicated class of models, such as hybrid inflation.

(5) Tensor modes. Inflation may generate tensor perturbations, which contribute to the CMB fluctuation for small ℓ. Excessive tensor modes cause the CMB harmonics increasing too rapidly towards a small ℓ. A reasonable guess for the limit from the current data is $r = T/S < 0.5$, but detail statistical analyses are yet to be carried out. Inflation that takes place significantly below the gravity scale does not generate tensor perturbations[90].

It would be instructive to impose our constraints on models of inflation. For example, chaotic inflation with the potential $V = m^2\phi^2$ (mass term only) predicts $n = 0.96$ just consistent with the upper limit of eq.(8). The value of the ϕ field at the epoch that the physical scale goes out of the horizon is $\phi_{\mathrm{phys}} = 2.8M_{pl}$ from (2). Inflaton mass $m = 2 \times 10^{13}$ GeV from (3). The model predicts $T/S = 0.12$, which is consistent with the observation.

There is a generic prediction of slow-roll inflation, $T/S \approx -6n_T$ with n_T the spectral index of the tensor mode. Unfortunately, this n_T is the quantity most difficult to measure. The relation

$$T/S \approx 6(1 - n) \tag{14}$$

often quoted in the literature holds only for specific classes of models. It is argued that most models of inflation predicts the relation either close to eq.(14) or $T/S \approx 0$ [91].

Many hundreds of inflation models have been considered by now[92]. I do not intend here to discuss model building, but it seems that there are no satisfactory models. I only briefly mention the outline of models which seem more generic and why they are not satisfactory; see [92] for details.

(1) ϕ^α potential. This is the prototype for chaotic inflation[93]. The slow roll condition and e-fold require that $\phi_{\rm phys}$ be larger than a few times $M_{\rm pl}$ and α be reasonably small. A red tilt $n<1$, and appreciable tensor perturbations that nearly satisfy eq.(14) are predicted. We must deal with super-Planck scale physics, which is beyond the understanding of particle physics today. The real problem, however, is that there is no principle to forbid higher order terms of the form $\phi^n/M_{\rm pl}^{n-4}$, which spoils the slow roll condition for $\phi \gtrsim M_{pl}$. An attempt to forbid such terms by introducing symmetry is presented at this Conference[94], but it lacks a particle physics motivation.

(2) $V_0[1-(\frac{\phi}{\mu})^p]$ type potential. This is typical of 'new inflation'[95]. Inflation starts with $\phi \simeq 0$, and ends with $\phi_{\rm end} \ll M_{pl}$. A red tilt is predicted, while tensor perturbations are very small due to a low energy scale involved. The difficulty is that one needs fine tuning for the initial condition. Furthermore, in most models of this type the inflaton field is not in thermal equilibrium, so that symmetry restoration at high temperature does not work. There is also a fundamental problem as to why universe has not collapsed long before the onset of this inflation.

(3) Hybrid inflation with two fields. The model is $V(\phi,\sigma)=\lambda(\sigma^2-M^2)^2+m^2\phi^2/2+g\phi^2\sigma^2/2$. This is a model which combines chaotic and new inflation features[96]. For ϕ greater than some critical value ϕ_c, $\sigma=0$ is the minimum and the model behaves as the chaotic type; ϕ remains large for a long time. At the moment when ϕ becomes smaller than ϕ_c, symmetry breaking occurs and rapid rolling of the field σ takes place. One nice feature with this model is that it can be embedded into SUSY or supergravity models, and the energy scale of the phase transition appears to agree with the unification scale[97]. The problems have been pointed out in more recent studies, however, that the model needs spatial homogeneity in the superhorizon scale in the preinflation era[98] and that the choice of the initial condition needs fine tuning to keep the ϕ filed in the desired valley[99]. This model predicts blue tilt $n>1$ in the tree level, but the tilt can be blue or red after loop corrections, depending on input parameters[92].

The general problem with inflation is a lack of satisfactory models motivated from particle physics. For example, such an idea that simply combines inflation with supergravity theory is liable to fail because the Kähler potential is too curved with the exponential dependence of the field. Most models discussed in the literature are constructed without regarding low energy physics; so the models are those just to do it for its own sake alone. For the view pint of astrophysical applications, the discovery that inflation does not exclude open universes[100] greatly diminished its predictive power. The observation of the tilt and the strength of the tensor mode will offer an important test for the model, though the current data are not yet accurate enough for this purpose. Astrophysicists may not feel comfortable, however, unless particle physics would explain why $V^{3/2}/V'$ takes a specific value as referred to in (3). A misprediction by an order of magnitude leads to a disaster for us (see below).

One philosophically interesting consequence arises from the fact that inflation never ends (eternal inflation) whichever inflation one considers[101]. This would result in different patches of the universe expanding differently; inflation leads to great inhomogeneity at superhorizon scales. Many universes are born at arbitrary instants in many different patches; after all we are living in just one of them and observe this 'small' patch as an 'entire universe'. This 'multiverse' picture would give a base to the speculation that many physical parameters may vary in different universes. In this picture the Big Bang is no longer given any special position.

8 Anthropic principle: use or misuse?

There are many constants that appear to be so tuned that they are just appropriate for the evolution of intelligent life. We would wonder whether what we expect to observe is restricted by the condition necessary for our presence as observers[85]. We have discussed that the vacuum energy is one such example. This is also true with the matter density. If the lightest neutrino would have mass larger than 10 keV, $\Omega_\nu \sim 100$ and the age of the universe would be too short for intelligent life to have developed. If, on the other hand, $\Omega < 0.01$, say, the galaxies would not have formed. In fact, $\Omega_{\rm CDM} \sim \Omega_b \sim \Omega_\nu \sim \lambda \sim O(1)$ up to only three orders of magnitude is an intriguing coincidence.

Another cosmologically important parameter is the strength of initial density fluctuations $Q \sim O(10^{-5})$. If this were larger by one order of magnitude, galaxies would be dominated by vast black holes; no stars or solar system could survive. If it were smaller by one order of magnitude, cooling does not efficiently work, and galaxies would not have formed[102]. From a view point of particle theory this is an obscure quantity $\sim V^{3/2}/V'$ in terms of the inflaton potential. Why this quantity takes a specific value which makes *us* habitable is puzzling.

There is a similar tricky coincidence (or providence) also in particle physics parameters, which is crucial to the evolution of intelligent life. The central issue is the parameters that affect element synthesis in the early universe and in stars. A small change of quark mass and/or the QCD strength stabilises or destabilises neutron, proton, deuterium, di-proton or di-neutron. Furthermore, the production of elements heavier than carbon just depends on the luck of the existence of a resonance in the ^{12}C system, which makes the bottleneck nuclear reaction $3\alpha \rightarrow ^{12}$C possible. A similar situation also exists with ^{16}O. Agrawal et al.[103] focused on the aspect that weak interaction scale is close to QCD scale rather than the Planck scale. Hogan[104] argued for the arrangement of mass difference among m_u, m_d and m_e, and a ± 1 MeV change of $m_d - m_u$ would disturb the existence of complex elements. He radically claims that the correct unification scheme should *not* allow calculation of $(m_d - m_u)/m_{\rm proton}$ from first principles. Rees[102] formulated the requirement in a way that fractional binding energy of helium, $\epsilon = {\rm BE}(^4{\rm He})/M(^4{\rm He})$, be tuned between 0.006 and 0.008.

It is clear that our existence hinges on delicate tuning of many parameters irrespective of whether it is a result of the anthropic principle or not. I refer the reader to Rees' book[102] for more arguments. Of course, the view on the anthropic principle is wildly divided. Hawking considers that why we are living in 3+1 dimension but not in 2+1 or 2+2 and why low energy theory is SU(3)×SU(2)×U(1) etc. are all results of the anthropic principle[105]. Physicists usually hope that all parameters are derived up to only one from fundamental principles, and the anthropic argument appears for them to be equivalent to giving up this effort. For instance, Witten[82] states that "I want to ultimately understand that, with all the particle physics one day worked out, life is possible in the universe because π is between 3.14159 and 3.1416. To me, understanding this would be the real anthropic principle ..." It is disappointing if the anthropic principle is the solution to many problems, but such a possibility is not excluded.

9 Baryogenesis

Before concluding this talk let me mention briefly baryogenesis, which is in principle in the interface between cosmology and particle physics. The real contact between the two disciplines, however, is a subject in the future: astrophysical cosmologists argue about an error of 10% for the baryon abundance, whereas particle physicists struggle to understand the order of magnitude.

Four major scenarios proposed so far and their state-of-the-art are:

(1) Grand unification. This prototype baryogenesis idea does not receive much support. Unless baryogenesis takes place with $B - L$ violated, the baryon excess is erased above the electroweak scale under the effect of Kuzmin-Rubakov-Shaposhnikov's (KRS) sphalerons. With the presence of inflation, whether the reheating temperature is sufficiently high to produce coloured Higgs is also a non-trivial problem. In SUSY GUT the reheat temperature T_R needs to be $> m_H^c \sim 10^{17}$ GeV, which is the lower limit on m_H^c to avoid fast proton decay. In supergravity theory the reheat temperature cannot be sufficiently high ($T_R < 10^9$ GeV) to avoid copious gravitino production.

(2) Electroweak baryogenesis with the KRS effect. The necessary condition is that the electroweak phase transition is of first order. Within the standard model this requires the Higgs mass to be lower than 70GeV, which is already much lower than the current experimental limit. The possibility is not yet excluded in supersymmetric extension. The electroweak transition can be strong if the stop mass is lower than the top quark mass[106]. A possible large relative phase between the vacuum expectation values of two Higgs doublets may bring CP violation large enough to give the observed magnitude of baryon number.

(3) Leptogenesis from heavy Majorana neutrino decay and the KRS mechanism. This mechanism works in varieties of unified models with massive Majorana neutrinos. For a recent review see [107]. Another mechanism is proposed for leptogenesis via neutrino oscillation[108].

(4) Affleck-Dine baryo/leptogenesis[109]. When the flat direction of the SUSY potential is lifted by higher-dimensional effective operator, coherent production of slepton and squark fields that carry baryon and lepton number takes place in the reheat phase of inflation. The oscillation starts and ends earlier than was thought in the original paper due to thermal plasma effect[110,111,112], leading to some suppressions of the baryon or lepton number production. Notwithstanding, this is still a viable scenario, although proper treatment of leptogenesis requires the lightest neutrino mass to be smaller than 10^{-8} eV for $T_R < 10^9$ GeV [112], the limit being stronger than was obtained in [110].

10 Conclusion

Over the last few years our understanding of cosmic structure formation based on the CDM dominance and statistical description has significantly tightened. The new CMB experiments reported this year further corroborated the validity of the model. Concerning the world model of the universe we may conclude that (1) open universes are excluded, (2) the EdS universe is excluded, and (3) a non-zero cosmological constant is present. These conclusions seem to be compelling in so far as we keep the current cosmic structure formation model. Note that the CDM model is the only model known today that successfully describes widely different observations. The cosmological parameters are converging to $H_0 = 62{-}83\,\mathrm{km\,s^{-1}Mpc^{-1}}$, $\Omega = 0.25 - 0.48$ and $\lambda = 0.75 - 0.52$.

For astrophysical cosmology an interesting problem is the baryon abundance. The CMB experiments indicate the optimum value of the baryon abundance higher than is inferred from nucleosynthesis by 50%, although the two are still consistent at a 95% confidence level. It is also interesting to notice that the dominant fraction of energy density of the universe is 'invisible' (see Table 1). The vacuum energy and CDM mass occupies over 95%. Even 3/4 of baryons are invisible. That visible with optical and X-ray is less than 1% of the total energy density.

I emphasise that the standard model of the universe involves three basic ingredients which are poorly understood in particle physics: (i) the presence of small vacuum energy ($\rho_V \simeq 3$ (meV)^4), (ii) the presence of cold dark matter, and (iii) the presence of scalar fields that cause inflation. We cannot have successful cosmology without these three substances.

Although the subjects I have discussed here serve as an interface between cosmology and particle physics, the particle physics part is still poorly understood for most aspects. More successful among others are speculation of candidate dark matter, and to some extent baryogenesis. At least we have a number of successful models which are related with low energy phenomenology; yet we cannot choose among these models. The attempt to understand inflation is much poorer: most models are constructed without regarding low energy phenomenology or even unified theories. Furthermore, theorists seem to assume too easily *ad hoc* mechanisms that are not internally motivated in order to solve 'difficulties' the own model create. Most attempts look like 'particle-physics-independent' models.

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TABLE 1
COSMIC ENERGY DENSITY BUDGET

entity		fraction		observation
vacuum		70%		invisible
CDM		26%		invisible
baryon		4%		
	warm gas		3%	invisible
	stars		**0.5%**	optical
	hot gas		**0.5%**	X-rays
neutrino		>0.1%		invisible

Note:–bold face means observable components

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